

Summer Assignment for AICE High Math I

This summer assignment is designed to help you maintain your skills in preparation for AICE High Math I. Nothing on this assignment is new, and your mastery of these topics is essential for success in this course. It is imperative that you understand how to complete each problem on your own.

****Neatly show all work and your solution for each problem on separate paper.****

Only scientific calculators are allowed, so the reliance on your graphing calculator is strongly discouraged. When exact answers are called for, your solutions should not contain decimals.

****You will have a quiz on this material shortly after school begins.****

This assignment and all associated work is due on or before September 6, 2019.

If additional help/review is needed, you can email Ms. Dempewolf at dempewjc@pwcs.edu and/or go online to the Khan Academy – Specifically Algebra 2, Trigonometry, Geometry and Physics topics may be helpful.

Algebra and Functions

1. **Factor completely:**

a. $4x^2 - 25$

b. $x^2 + 7x + 6$

c. $x^3 - 8$

d. $x^3 + 8$

e. $27x^3 - 125y^3$

f. $x^4 + 11x^2 - 80$

g. $x^3 - xy^2 + x^2y - y^3$

2. **Solve for x:**

a. $\frac{1}{x^4} - \frac{7}{x^2} + 10 = 0$

b. $x - 3x^{\frac{1}{2}} + 2 = 0$

c. $2x + 5\sqrt{x} - 12 = 0$

d. $\frac{1}{(x^2-4)^2} - \frac{1}{x^2-4} - 20 = 0$

e. $8x^4 - 4x^2 = 0$

f. $3x^{\frac{2}{3}} - x^{\frac{1}{3}} - 4 = 0$

3. Use the discriminant of the quadratic equation to answer the following questions

a. How many roots do the following quadratics have?

i. $x^2 - 4x - 7$

ii. $3x^2 + 5x + 1$

iii. $2 + x - x^2$

iv. $7 + 2x - 5x^2$

b. $3x^2 + kx + 12 = 0$ has two solutions. Find the possible values of k .

c. $x^2 - 3x + a = 0$ has two equal roots. Find the possible values of a .

d. $x^2 + px + (p - 1) = 0$ has no solutions. Find the possible values of p .

4. Find the point(s) of intersection of the following lines and curves. Show all algebra used.

a. $\begin{cases} x - y = 2 \\ 2x^2 - 3y^2 = 15 \end{cases}$

b. $\begin{cases} 2x - y = 1 \\ 3x^2 - y^2 = 3 \end{cases}$

c. $\begin{cases} 4x + y = 2 \\ 4x + y^2 = 8 \end{cases}$

d. $\begin{cases} x + y = 1 \\ x^2 - xy = 15 \end{cases}$

e. $\begin{cases} xy + x = -3 \\ 2x + 5y = 8 \end{cases}$

5. Simplify (write all final answers with positive exponents and roots as applicable):

a. $5^4 \times 5^3$

b. $(3^2)^4$

c. $-3x^{-3}$

d. $-5\left(\frac{3}{2}\right)(4 - 9x)^{-\frac{1}{2}}(-9)$

e. $2\left(\frac{2}{2-x}\right)\left(\frac{-2}{(2-x)^2}\right)$

f. $(16x^2y)^{\frac{3}{4}}$

g. $\frac{\sqrt{4x-16}}{\sqrt{(x-4)^2}}$

h. $-4\left(\frac{2x-1}{2x+1}\right)^{-3}\left[\frac{2(2x+1)-2(2x-1)}{(2x+1)^2}\right]$

i. $\frac{\frac{1}{2}(2x+5)^{-\frac{3}{2}}}{\frac{3}{2}}$

6. Solve the following inequalities:

a. $|x - 3| > 12$

b. $|x - 3| \leq 4$

c. $|10x + 8| > 2$

d. $|3x - 4| \geq -2$

e. $|x - 6| < -8$

7. The graphs of eight functions are sketched below. Match the graphs with the appropriate equation.

a. $y = 5 - 3x$

b. $y = \frac{12}{x}$

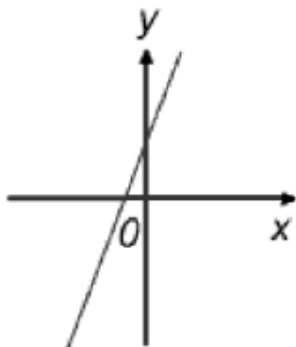
c. $y = 2x^3$

d. $y = 4 + 2x$

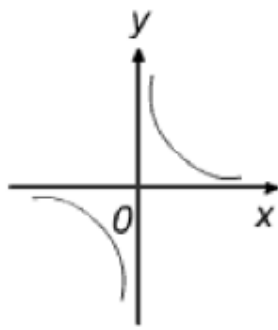
e. $y = -3x^3$

f. $y = \frac{2}{x^2}$

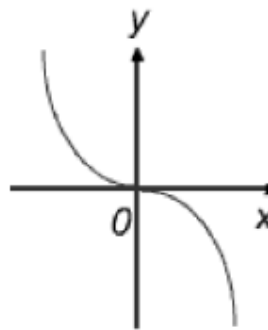
g. $y = x^3 + x^2 - 12x$



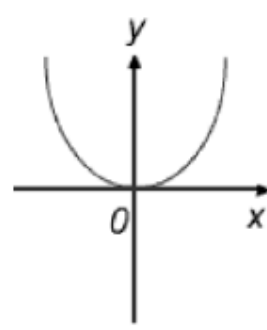
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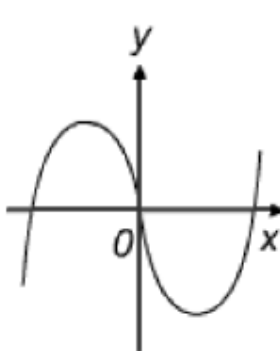
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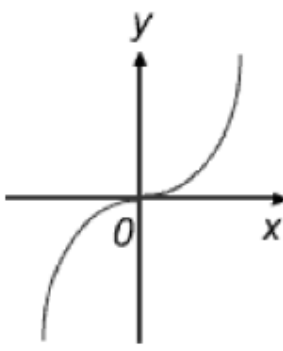
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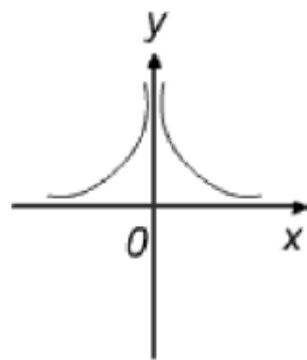
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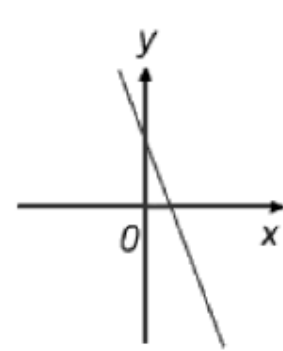
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8. Simplify completely:

a. $\frac{\frac{1}{x}+4}{\frac{1}{x}-2}$

d. $\frac{1-\frac{2}{3x}}{x-\frac{4}{9x}}$

g. $\frac{\frac{2}{1-x}+\frac{1+x}{x}}{\frac{x}{x}+\frac{1}{1+x}}$

b. $\frac{x-\frac{1}{x}}{x+\frac{1}{x}}$

e. $\frac{\frac{x^2-y^2}{xy}}{\frac{x+y}{y}}$

h. $\frac{\frac{4}{x-5}+\frac{2}{x+2}}{\frac{2x}{x^2-3x-10}}+3$

c. $\frac{\frac{3}{x}-\frac{4}{y}}{\frac{4}{x}-\frac{3}{y}}$

f. $\frac{x^{-3}-x}{x^{-2}-1}$

9. Let P(-3, 1) and Q(5, 6) be two points in the coordinate plane.

- Find the distance between P and Q.
- Find the midpoint of the segment PQ.
- Find the slope of the line that contains P and Q.
- Find the equation of the perpendicular bisector of the line that contains P and Q.
- Find an equation for the circle for which the segment PQ is a diameter.

10. Repeat steps a-d from problem number 9 for the points A(-2, 4) and B(3, 10).

11. If $f(x) = x^2 + 1$ and $g(x) = x - 3$, find and **simplify** the following

- $f(g(x))$
- $g(f(x))$
- $f(g(2))$
- $g(f(2))$
- $g(g(g(x)))$

12. Write $6^{2x} = 25$ in logarithmic form

13. Write the equation $\ln A = 3$ in exponential form.

14. Use the laws of logarithms to expand the expression: $\log_3 \sqrt[3]{\frac{x+2}{x^4(x^2+4)}}$

15. Combine into a single logarithm: $\ln x - 2 \ln(x^2 + 1) + \frac{1}{2} \ln(3 - x^4)$

16. If $f(x) = x^2 - 1$, describe, **in words**, what the following operations would do to the graph of $f(x)$:

- $f(x) - 4$
- $f(x - 4)$
- $-f(x + 2)$
- $5f(x) + 3$
- $f(2x)$
- $|f(x)|$

17. Write down the expression to find the n^{th} term for each of the following sequences.

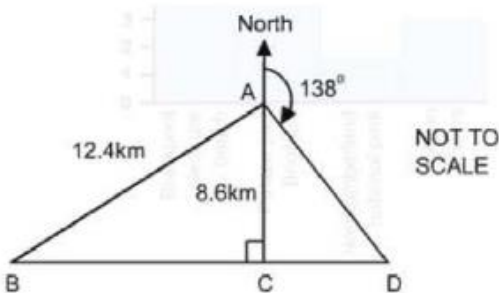
- 2, 5, 10, 17, 26, 37, ...
- 4, 16, 36, 64, 100, 144, ...

18. Find the first six terms and the sixth partial sum of the sequence whose n^{th} term is $a_n = 2n^2 - n$

19. An arithmetic sequence begins 2, 5, 8, 11, 14, ...
- Find the common difference, d
 - Find the formula for the n^{th} term of the sequence
 - Use your formula to find the 35th term of the sequence.
20. A geometric sequence begins 12, 3, $\frac{3}{4}$, $\frac{3}{16}$, $\frac{3}{64}$, ...
- Find the common ratio, r
 - Find the formula for the n^{th} term of the sequence
 - Use your formula to find the 10th term of the sequence.
21. The first term of a geometric sequence is 25 and the fourth term is 0.2.
- Find the common ratio, r , and the fifth term.
 - Find the partial sum of the first eight terms.
22. The first term of an arithmetic sequence is 10 and the tenth term is 2.
- Find the common difference, d , and the 100th term of the sequence.
 - Find the partial sum of the first ten terms.

Trigonometry

23. The diagram shows the position of points A, B, C and D. A is due North of C. The straight line BCD is perpendicular to AC. A is 12.4 km from B and 8.5 km from C.



- Calculate the distance BC.
 - The bearing of D from A is 138° . Calculate the distance DC.
24. If point P is on the terminal side of θ , find all six trigonometric functions of θ . Draw a picture for each.
- $P(-2, 4)$
 - $P(\sqrt{5}, -2)$
25. If $\cos \theta = \frac{5}{13}$, and θ is in quadrant II, find $\sin \theta$ and $\tan \theta$
26. If $\cot \theta = 3$ and θ is in quadrant III, find $\sin \theta$ and $\cos \theta$

27. Find the exact value of the following **WITHOUT A CALCULATOR!!!!**

a. $\sin^2 225^\circ - \cos^2 225^\circ$

b. $(6 \sec 180^\circ - 4 \cot 90^\circ)^2$

c. $(4 \cos 30^\circ - 6 \sin 120^\circ)^{-2}$

28. Solve the following triangles completely accurate to 3 decimal places. (Drawing a picture always helps.)

a. $B = 16^\circ, C = 90^\circ, a = 21.7$ cm

b. $C = 92^\circ, a = 6$ ft, $c = 95$ in

29. Solve each equation on the interval $[0, 2\pi)$ **WITHOUT A CALCULATOR**. Report exact answers when possible. (Note: don't divide by a trig function or you will lose valid solutions)

a. $\sin x = \frac{1}{2}$

b. $\cos^2 x = \cos x$

c. $2 \cos x + \sqrt{3} = 0$

d. $4 \sin^2 x = 1$

i. $8 \cos^2 x - 2 \cos x = 1$ (you can use a calculator to finish this one)

e. $2 \sin^2 x + \sin x = 1$

f. $\cos^2 x + 2 \cos x = 3$

g. $2 \sin x \cos x + \sin x = 0$

h. $\sin^2 x - \cos^2 x = 0$

30. Verify each identity:

a. $\tan \theta \sin \theta + \cos \theta = \sec \theta$

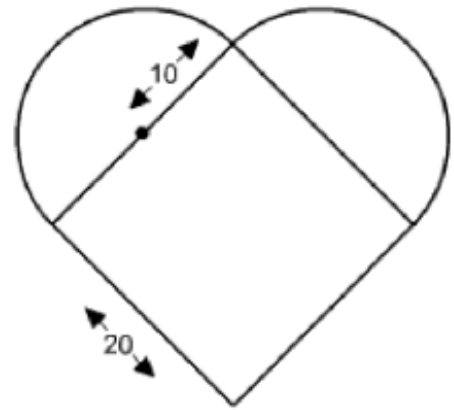
b. $\frac{\tan \theta}{1 - \cos \theta} = \csc \theta (1 + \sec \theta)$

c. $\frac{1 + \sin x}{\cos x} + \frac{\cos x}{1 + \sin x} = \frac{2}{\cos x}$

Geometry

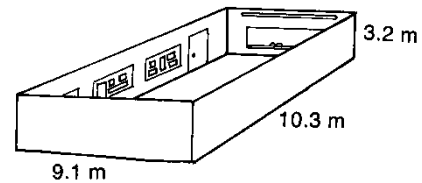
31. Instructions for making a heart shaped cake:

- Bake a square cake of side 20 cm.
- Bake a round cake of radius 10 cm.
- Cut the round cake in half and join the two halves to the square cake as shown.
 - Find the area of the heart shape (don't forget to label with appropriate units)
 - A red ribbon is fixed around the sides of the heart shaped cake with ends overlapping by 3 cm. Find the length of the ribbon required.
 - Suppose the finished cake is 4 cm tall. What is the volume of the cake?

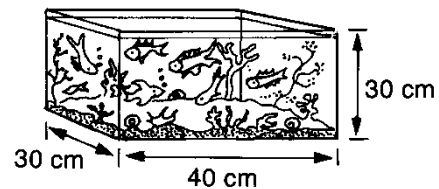


32. A classroom has the dimensions shown. Find:

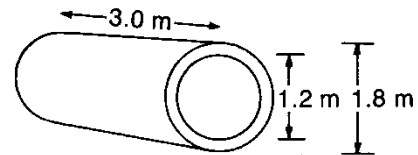
- the area of the floor
- the volume of the room
- the total area of the four walls.



33. If one guppy requires 5 L of water to live happily, what is the maximum number of guppies that should be kept in this aquarium?



34. A section of concrete pipe 3.0 m long has an inside diameter of 1.2 m and an outside diameter of 1.8 m. What is the volume of concrete in this section of pipe?



35. A cereal company decided to make an odd-shaped box for a promotion they are doing. The new design is a rectangular prism with length of 10 in., width of 8 in., and height of 4 in. and attached to the rectangular prism is a cylinder with a radius of 2 in. and a height of 10 in. How much cereal will fit in the box?

Mechanics

Calculate displacement (distance from start), velocity, and acceleration using the following formulas. *Note: displacement is in meters, velocity is in m/s, and acceleration is m/s².*

When velocity is constant → Displacement = velocity × time

When acceleration is constant → Velocity = acceleration × time

36. Find the time taken for a particle traveling
 - a. 30 m at a velocity of 5 m/s
 - b. 8 m at a velocity of 0.2 m/s
 - c. 5 m at a velocity of 25 m/s
37. Find the displacement of a particle travelling with
 - a. A velocity of 12 m/s for 12 seconds
 - b. A velocity of 0.4 m/s for 10 seconds
 - c. A velocity of 30 m/s for 0.5 seconds
38. Find the velocity of a particle that has
 - a. A displacement of 24 m in 8 seconds
 - b. A displacement of 45 m in 30 seconds
 - c. A displacement of 10 m in 50 seconds
39. Find the change in velocity when a particle accelerates at
 - a. 10m/s² for 10 seconds
 - b. 0.2 m/s² for 30 seconds
40. Find the acceleration when a particle's velocity changes
 - a. From 20 m/s to 50 m/s in 10 seconds
 - b. From 44 m/s to 43 m/s in 6 seconds
41. Find the time taken for a particle to accelerate from 15 m/s to 60 m/s at 15 m/s².