SUMMER ASSIGNMENT FOR AICE High Maths 2

Due September 6th

This summer assignment is designed to prepare you for AICE High Maths 2. Nothing on the summer assignment is new. Everything is a review of topics students learned in IGSCE Geometry, IGSCE Algebra II/Trig and AICE High Maths 1. If you want to be successful during AICE High Maths 2, you *must* be able to understand and apply this information throughout next year. The assignment may be completed with another student but be certain that YOU understand how to complete every problem.

- **Neatly show all work for each problem on a separate sheet of paper.**
- *Scientific Calculators only on this assignment.*

Probability and Statistics

21. In the game of "Soap," two fair dice, with the faces numbered 1 to 6 are thrown. The total scores on the dice is the score for that turn. The player moves the same number of places as their score.
    a. Khalid wants to land on the space marked "Albert Square." He is now on "Coronation Street" which is 6 spaces away. What is the probability that Khalid lands on "Albert Square" on his next turn.
    b. Caroline does not want to land on "Ramsey Street" which is 7 spaces away. What is the probability that Caroline does not land on "Ramsey Street?"
    c. You can only escape "Cell Block H" if you score the same number on each die (aka, doubles). John is on "Cell Block H." What is the probability that he escapes on his next turn?

22. In a class of 20 pupils, 11 have dark hair, 7 have fair hair and 2 have red hair. Two pupils are chosen at random to collect the homework. What is the probability that they
    a. both have fair hair?
    b. each have hair of a different color?

23. The Morgan family leave Manchester to catch the 12 noon ferry from Dover. The probability that they will catch the ferry is 0.9. The Collins family leaves Croydon to catch the same ferry. The probability that they will catch the ferry is 0.8. The two events are independent. Find the probability that
    a. Both families will catch the ferry
    b. Neither will catch the ferry

24. A bag contains five discs that are numbered 1-5. Rachel takes a disc from the bag. She notes the number and puts the disc back. She shakes the bag and picks again. She adds this number to the first number.
    a. Complete the table to show all possible totals.

First number

Second number	+	1	2	3	4	5
	1	2				
	2					
	3				7	
	4					
	5					

    b. Find the probability that Rachel's total is
        i. 10
        ii. 1
        iii. 3 or 4

25. Anil has five bars of chocolate in a cupboard. Three are Kit-Kats, one is a Mars bar and one is a Fudge bar. He takes one at random on each weekday to eat at school.
    a. Calculate the probability that the bar of chocolate will be a Kit-Kat on both Monday and Tuesday of that week.
    b. Calculate the probability that the bar of chocolate will be a Kit-Kat on Monday, Tuesday, and Wednesday and a Mars bar on Thursday.
    c. Calculate the probability that the bar of chocolate will not be a Kit-Kat on any two consecutive days in that week.

26. The heights of 100 plants were measured. The results are shown in the table below.

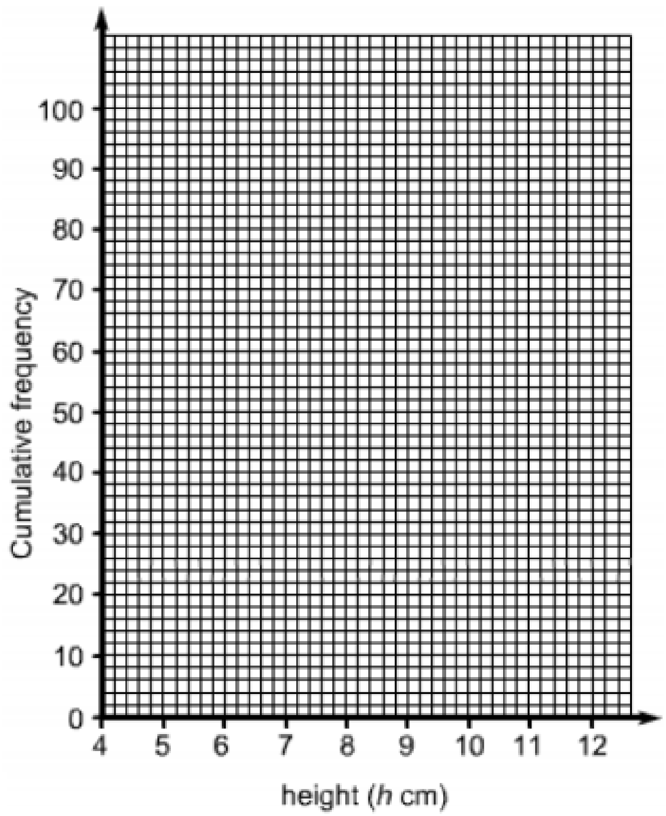
Height (cm)	$4 < h \le 5$	$5 < h \le 6$	$6 < h \le 7$	$7 < h \le 8$	$8 < h \le 9$	$9 < h \le 10$	$10 < h \le 11$	$11 < h \le 12$
Frequency	3	7	11	28	24	17	8	2

a.

b. Complete the **cumulative** frequency table for the 100 plants.

Height (cm)	$h \le 4$	$h \le 5$	$h \le 6$	$h \le 7$	$h \le 8$	$h \le 9$	$h \le 10$	$h \le 11$	$h \le 12$
Cumulative Frequency	0	3							

c. Draw the cumulative frequency diagram on the grid below.



d. Use your graph to estimate how many plants are less than 9.4 cm high.

e. Use your cumulative frequency diagram to estimate the inter-quartile range of the heights.

27. Over the past year, John as purchased 30 books.

   a. In how many ways can he pick four of these books and arrange them on his nightstand bookshelf?

   b. In how many ways can he choose four of these books to take with him on vacation at the shore?

28. A hospital cafeteria offers a fixed-price lunch consisting of a main course, a dessert, and a drink. If there are four main courses, three desserts, and six drinks to pick from, in how many ways can a customer select a meal consisting of on choice from each category?

Pure Maths:

1 The term independent of x in the expansion of $\left(2x + \frac{k}{x}\right)^6$, where k is a constant, is 540.

(i) Find the value of k. [3]

(ii) For this value of k, find the coefficient of x^2 in the expansion. [2]

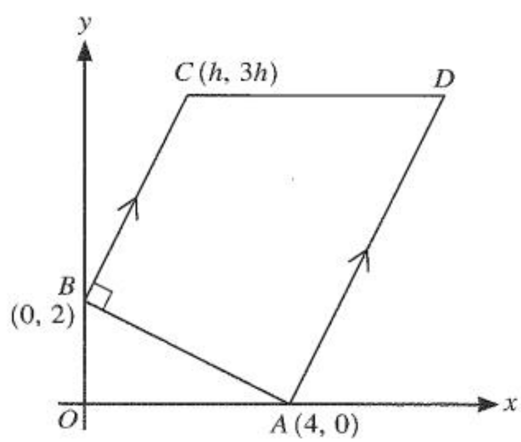
2 The line $4y = x + c$, where c is a constant, is a tangent to the curve $y^2 = x + 3$ at the point P on the curve.

(i) Find the value of c. [3]

(ii)Find the coordinates of P. [2]

3 A sector of a circle of radius r cm has an area of A cm^2. Express the perimeter of the sector in terms of r and A. [4]

4



The diagram shows a trapezium $ABCD$ in which the coordinates of A, B and C are (4, 0), (0, 2) and $(h, 3h)$ respectively. The lines BC and AD are parallel, angle $ABC = 90°$ and CD is parallel to the x-axis.

(i) Find, by calculation, the value of h. [3]

(ii) Hence find the coordinates of D. [3]

5 The function f is defined by $f(x) = -2x^2 + 12x - 3$ for $x \in \mathbb{R}$.

(i) Express $-2x^2 + 12x - 3$ in the form $-2(x + a)^2 + b$, where a and b are constants. [2]

(ii) State the greatest value of $f(x)$. [1]

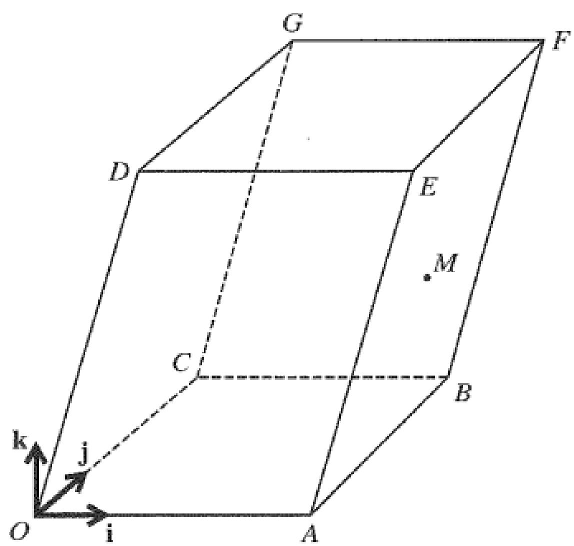
The function g is defined by $g(x) = 2x + 5$ for $x \in \mathbb{R}$.

(iii) Find the values of x for which $gf(x) + 1 = 0$. [3]

6 **(i)** Prove the identity $\left(\frac{1}{\cos x} - \tan x\right)^2 \equiv \frac{1 - \sin x}{1 + \sin x}$. [4]

(ii) Hence solve the equation $\left(\frac{1}{\cos 2x} - \tan 2x\right)^2 = \frac{1}{3}$ for $0 \leqslant x \leqslant \pi$.

7



The diagram shows a three-dimensional shape in which the base $OABC$ and the upper surface $DEFG$ are identical horizontal squares. The parallelograms $OAED$ and $CBFG$ both lie in vertical planes. The point M is the mid-point of AF.

Unit vectors $\mathbf{i}$ and $\mathbf{j}$ are parallel to OA and OC respectively and the unit vector $\mathbf{k}$ is vertically upwards. The position vectors of A and D are given by $\overrightarrow{OA} = 8\mathbf{i}$ and $\overrightarrow{OD} = 3\mathbf{i} + 10\mathbf{k}$.

(i) Express each of the vectors $\overrightarrow{AM}$ and $\overrightarrow{GM}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [3]

(ii) Use a scalar product to find angle GMA correct to the nearest degree. [4]

8 **(a)** The third and fourth terms of a geometric progression are 48 and 32 respectively. Find the sum to infinity of the progression. [3]

(b) Two schemes are proposed for increasing the amount of household waste that is recycled each week.

Scheme A is to increase the amount of waste recycled each month by 0.16 tonnes.

Scheme B is to increase the amount of waste recycled each month by 6% of the amount recycled in the previous month.

The proposal is to operate the scheme for a period of 24 months. The amount recycled in the first month is 2.5 tonnes.

For each scheme, find the total amount of waste that would be recycled over the 24-month period. [5]

Scheme A ....

Scheme B ....

9 The function f is defined by $\mathrm{f}(x) = 2 - 3\cos x$ for $0 \leqslant x \leqslant 2\pi$.

(i) State the range of f. [2]

(ii) Sketch the graph of $y = \mathrm{f}(x)$. [2]

The function g is defined by $\mathrm{g}(x) = 2 - 3\cos x$ for $0 \leqslant x \leqslant p$, where p is a constant.

(iii) State the largest value of p for which g has an inverse. [1]

(iv) For this value of p, find an expression for $\mathrm{g}^{-1}(x)$. [2]

10 A curve for which $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2x - 5$ has a stationary point at (3, 6).

(i) Find the equation of the curve. [6]

(ii) Find the x-coordinate of the other stationary point on the curve. [1]

(iii) Determine the nature of each of the stationary points. [2]

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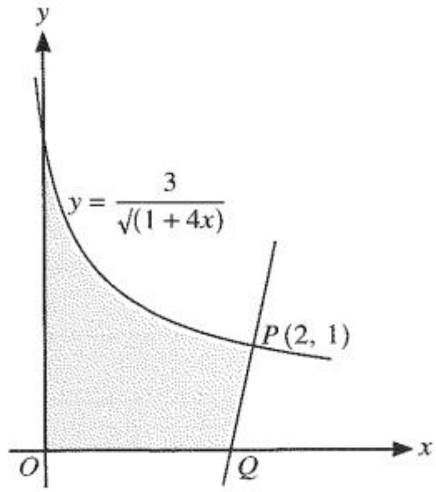

The diagram shows part of the curve $y = \frac{3}{\sqrt{(1 + 4x)}}$ and a point $P\,(2,\ 1)$ lying on the curve. The normal to the curve at P intersects the x-axis at Q.

(i) Show that the x-coordinate of Q is $\frac{16}{9}$. [5]

(ii) Find, showing all necessary working, the area of the shaded region. [6]